

Analysis of Contact Stresses at Trochoidal Gearing by Numeric Approach

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ABSTRACT: In this paper the analysis of contact stresses at trochoidal gearing of the rotational elements with fixed shafts and driving rotor that is connected with internal element is presented. At first, the basic characteristics and acting mechanisms of those pumps are presented in the paper. Also, the review of development and state of the art in the area of numerical analysis of gerotor pumps are presented. In the aim to determine the maximal stresses, in the second part of the paper, numerical analysis of contact stresses that are consequence of torque transmission from driving internal element to driven external gear is described. Graphical presentation of contact stresses values determined by numerical and analytical approach that is done, is presented in this part of the paper, in order to evaluate the differences and deviations of those values.

Key words: gerotor pumps, rotational elements, contact stresses, numerical analysis

I. INTRODUCTION

Determination of stress-strain state at elements of mechanical constructions is important step in the process of constructional optimization, so as improvement of its safety and reliability. The gears are basic components of machine systems that safety and reliability highly dependent of their properties. Utilization of gears with involute gearing besides significant technical and economic benefits comes with certain problems (relatively low load capacity of tooth sides and high contact stresses) that cause reduction of efficiency. Simultaneously with development of rotational motors, development of pumps with trochoidal gearing is done, based on kinematic principle of planetary mechanisms known as gerotor. Application of trochoidal gearing at present constructions can provide adequate efficiency and exploitation life altogether with minimal dimensions and weight. Trochoidal profile of gearing is applied at cyclo reducer special group of planetary gear transmitters and at large number of rotational machines with various applications, such as: rotational pumps, rotational motors, compressors and blowers. The numerical analysis of contact stresses at rotational elements with trochoidal profile of internal gearing by finite elements method is done in this paper. The comparison of values determined by numerical method and analytically assumed is done and evaluated. Also, dependence of contact stresses at certain gear teeth on value of rotational angle of external gear. At the area of research of gerotor pumps, Gamez presented analysis of geometric and functional properties of gerotor pumps. By application of modified Colbourne method he calculates contact stresses upon which he suggest optimal constructional solution of considered pump. The verification of results obtained by analytical approach were done by experimental testing by photoelastic method [1, 2]. Ivanović give methodology for identification of optimal form of trochoidal profile of tooth gearing at rotational (gerotor) pump, so as develop analytical method for calculation of maximal contact stresses at rotational elements of those pumps. The presented methodology is illustrated at actual pump and obtained solution is proposed as optimal for considered exploitation factors [3]. Fabiani with associates presented modeling and simulation of gerotor pump for lubrication of internal combustion engine [4].

II. FACTORS FOR NUMERICAL ANALYSIS THAT ARE ANALYTICALLY CALCULATED AND ASSUMED

2.1. Calculation of contact point coordinates

As contacts of trochoidal profiles are present at all teeth simultaneously, it is necessary to determine general relation for coordinates of points on all gear teeth profiles. Basic geometrical dimensions and relations that are involved in calculations are presented at Fig. 1.

Where is:

$x_f O_f y_f$ – fixed coordinate system at the center of internal gear

$x_i O_i y_i$ – coordinate system attached to the internal gear, $x_a O_a y_a$ – coordinate system attached to the external gear,

e – center distance of internal and external gear (pump eccentricity),

r_c – radius of the equidistant,

r_i – radius of the internal gear pitch circle,

r_a – radius of the external gear pitch circle, C – pitch point,

i – order of contact point, δ_i – involute angle,

φ_i – rotation angle of the internal gear about its own axis,

φ_a – rotation angle of the external gear about its own axis,

ψ – referent rotation angle and

d – distance between the generating point D and the center of the external gear.

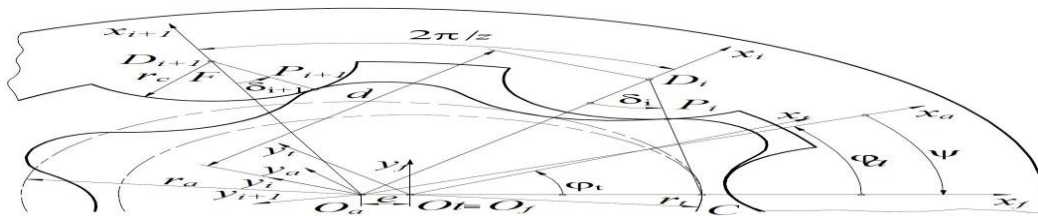


Fig.1. Schematic presentation of gear coupling at trochoidal pump with geometric dimensions [3]

On the basis of the geometrical properties presented at Fig. 1, the general equitation that determine coordinates of contact point at coupled profiles in coordinate system of the trochoide [3]:

$$r_{pi} = \begin{matrix} x_{pi}^{(i)} \\ y_{pi}^{(i)} \end{matrix} = \begin{matrix} z \lambda \cos \frac{\pi 2i-1}{z} - y_i - l \cos \frac{\pi 2i-1}{z} - \psi + \epsilon \\ z \lambda \sin \frac{\pi 2i-1}{z} - \psi - c \sin \frac{\pi 2i-1}{z} - \psi + \epsilon \end{matrix} \quad (1)$$

Where are:

r_{pi} – radius vector of contact point with i order,

$x_{pi}^{(i)}$ – x coordinate of contact point with i order, $y_{pi}^{(i)}$ – y coordinate of contact point with i order,

z – number of teeth on external gear, λ – trochoid coefficient,

i – order of contact point and

c – equidistant coefficient, ($c = \frac{r}{\epsilon}$)

2.2 Adopted values and calculation of torque

On the basis of the calculated resultant torque by analytical method - T (torque that is determined as difference between driving torque and torque of pressure forces of fluid at external surface of internal gear), the values presented at Tab. 1 are obtained [3].

Table 1. Adopted values of torque

| | $\lambda=1.575$ | $\lambda=1.375$ |
|-----------------------|-----------------|-----------------|
| $\varphi_a, [^\circ]$ | $T, [Nm]$ | $T, [Nm]$ |
| 0 | 0.632 | 0.621 |
| 15 | 0.290 | 0.352 |
| 25 | 0.041 | 0.059 |
| 45 | 0.290 | 0.352 |
| 60 | 0.632 | 0.621 |

III. CALCULATION OF STRESSES BY FINITE ELEMENT METHOD (FEM)

Finite element method is method of solving engineering problems and problems of mathematical physics. The finite element method is used for structural analysis, contact phenomena, solving of temperature fields, fluid stream problems and mass transport problems. Modeling process consists of continuum discretization (body or structure). Discretized model is formed of finite elements, related at nodes (linear elements), at common lines (plain elements) or at common surfaces (3D elements). The basic idea of finite elements method application is verification of analytical method that is designed for analysis of distribution of contact stresses at gear teeth in mash. Numerical model, used for research and evaluating of considered values, is developed according to assumptions and generalizations that are done on the basis of theoretical approach. At the definition of problems that are solved by finite element method, the following assumptions have to be done: axis of gears are fixed, rotations are transmitted from internal to external gear, load at present contact line of gear teeth is constant, problem is considered as static one, physical properties of material are constant, thermal effects are not considered and friction effect are neglected. Simulations of load (contact stresses) are done by finite elements method at gear of gerotor pump type GP-575 (gerotor with trochoide coefficient $\lambda=1.575$) and GP-375 (gerotor with trochoide coefficient $\lambda=1.375$), presented at Fig.2. Software package FEMAP v10.3.0 that is primarily designed for structural analysis [5].

IV. NUMERICAL CALCULATIONS OF CONTACT STRESSES, ITS VISUALISATIONS AND EVALUATIONS

In order to numerical calculation be done, it was necessary to repeat procedure of structural analysis several times for different values of lead angle φ_a . Analysis is done for different angular positions of external gear, starting from 0° to 60° ($0^\circ, 15^\circ, 25^\circ, 45^\circ$ and 60°), due to symmetrical distribution of results regarding to positions that relate to one rotation of pump mean one

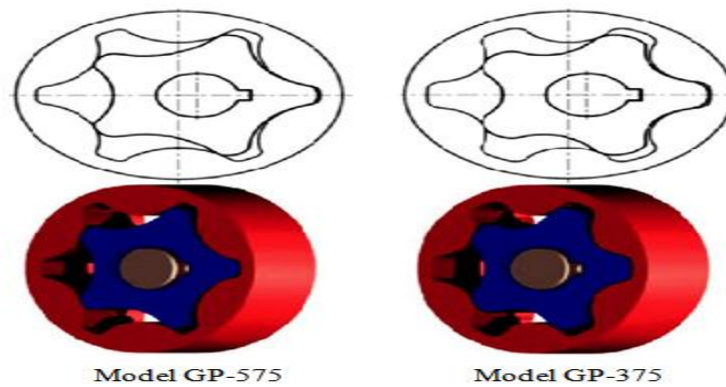


Fig.2. Considered pump models

working cycle [5]. The visualization of analyzed model with equivalent (Von Mises) stresses at position that is related to angle of rotation of external gear $\varphi_a=0^\circ$ for considered gerotor pumps GP-575 and GP-375 are presented at Fig.3 and Fig.4, respectively.

On the basis of the numerical calculations and determined maximal stresses at contact zones of gears in mash of considered types of gerotor, analysis can be done in order to light stress states. Values of maximal stresses at contact

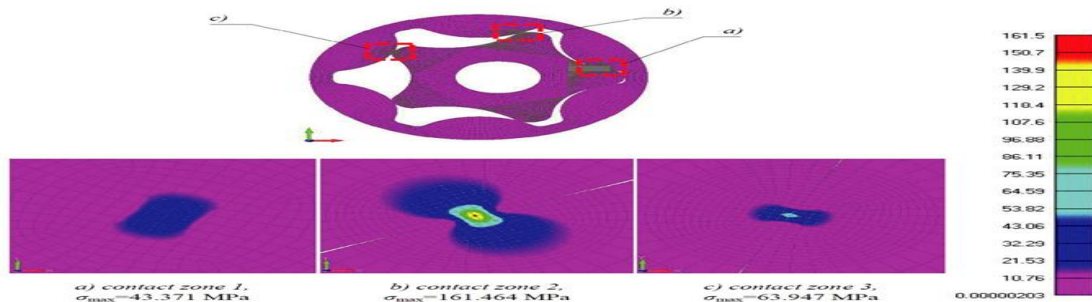


Fig.3. Visualization of contact stresses distribution at different zones at model GP-575 for $\varphi_a=0^\circ$ with maximal values

zones 1, 2 and 3 are peak at considered construction, so its calculations and analysis are of the highest

important to verify analytically obtained values at those positions.

In order to simplified consideration, the comparative presentation of distribution and values of maximal stresses at contact zones of gears, calculated by analytical and numerical method for considered types of gerotor are done at Fig.5.

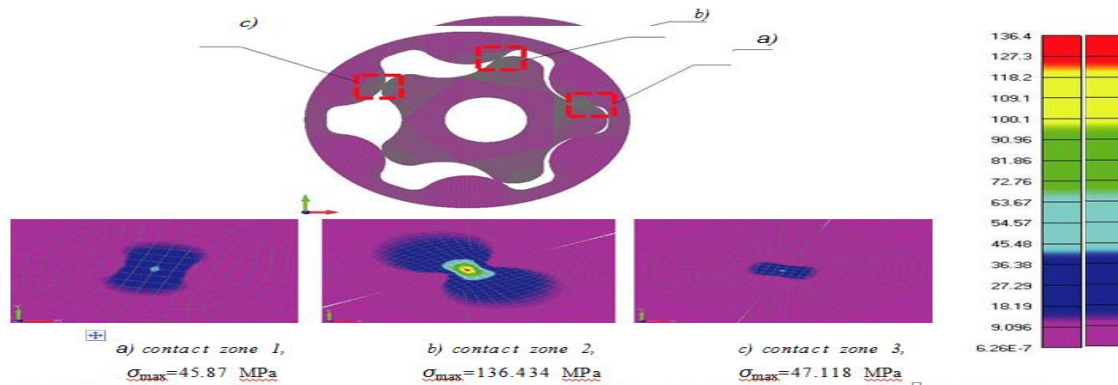


Fig.4. Visualization of contact stresses distribution at different zones at model GP-375 for $\varphi_a=0$ with maximal values

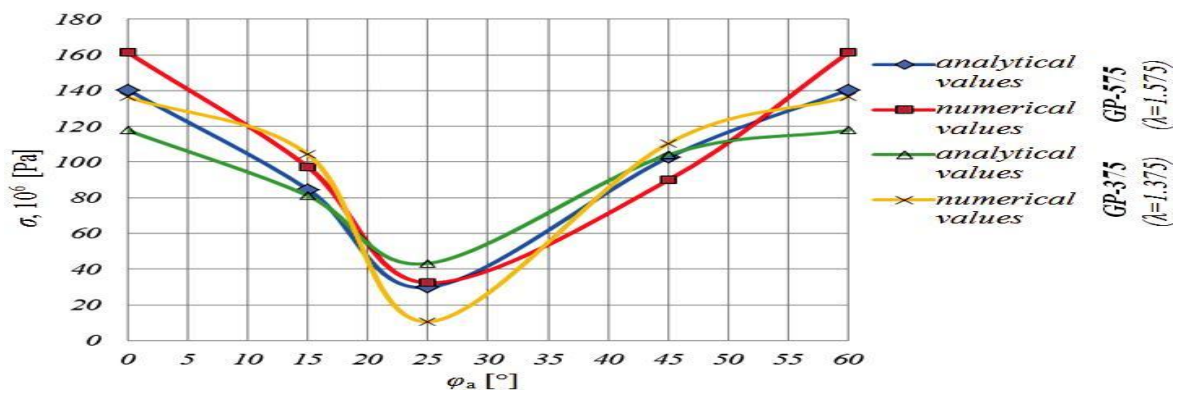


Fig.5. Results of analytical and numerical calculation of maximal stresses in relation to angle of rotation of external gear

Presented values of stresses at both considered types of gerotor, obtained by numerical and analytical method, have similar trend, so it can be concluded that numerical model and calculation procedure are adequate. In order to consider deviation of stress values, obtained analytically and numerically, comparative analysis based on values of relative deviation in % calculated by following relation:

Where is:

$\Delta\sigma$ – relative deviation of considered value,

σ – higher value of stress (obtained by analytical or numerical method) and

σ^* - lower value of stresses.

Comparative presentation of relative deviation obtained by numerical and analytical method, for considered types of gerotor, GP-575 and GP-375, are given at Fig.6 and Fig.7, respectively.

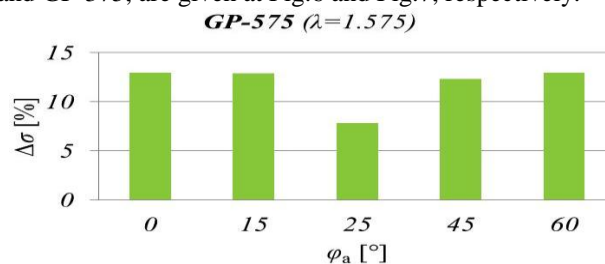


Fig.6. Relative percentage deviation of maximal stresses for different values of rotational angle of external gear in GP-575 ($\lambda=1.575$)

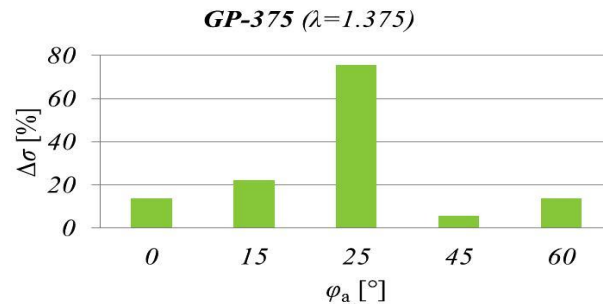


Fig.7. Relative percentage deviation of maximal stresses for different values of rotational angle of external gear in GP-375 ($\lambda=1.375$)

Relative deviation of maximal stresses, obtained by analytical and numerical method, at models of considered types of gerotor with same dimensions of finite elements for different angles of rotation of external gear, point out that numerical calculations are reliable for some angles of rotation (0° , 15° , 45° and 60°). As consequence of different shape of surface at zone around contact point for angle of rotation $\varphi_a=25^\circ$, significant deviation is caused. As value of maximal stresses obtained by numerical method have significant deviation only for abovementioned angle, it can be concluded that calculation of stresses by numerical method have slightly deviation from analytically obtained one, means that stresses are inside limits. As relative deviations are insignificant, results obtained by this method are relevant. The obtained results by this method verified lower values of contact stresses by analytical method for gerotor type GP-375 and can be reliably used for calculation of stresses at elements with complex geometry.

V. CONCLUSION

On the basis of the analysis and evaluation of the obtained results the following general conclusions can be done: Diagrams of stresses, obtained by analytical and numerical calculations, have similar trends. This means that used models for numerical calculations are relevant, so those models can be used for future research and analysis in this area for identification of constructional solution with minimal stresses. Values of maximal stresses obtained by numerical calculations are higher than related values obtained by analytical calculations. Maximal value of stresses act at positions that are related to angles $\varphi_a=0^\circ$ and $\varphi_a=60^\circ$. On the basis of those conclusions it can be stated that both analytical and numerical method provides relevant results for the analysis of stress values sensitivity on variation of input parameters. The special focus must be put on problems of motion and position of internal and external gear in exploitation. Those positions and motions can cause: wear and failures of elements, reduce its working life, reliability and so on. Influences of input parameter variations to maximal stresses at contact zones are significant. The presented analysis provides optimizations of constructions and design of elements shapes. Those optimal constructional solutions prevent possible failures and wear at elements during exploitation under maximal working pressure and critical rotation angles, $\varphi_a=0^\circ$ and $\varphi_a=60^\circ$. Further evaluation of the results and development of analysis, that are presented in this paper, can be pointed to variations of parameters, discretization by different type of finite elements. The analysis of the influence of finite elements dimensions to calculation results also can be of future interest. The primary aim is identification of gerotor design with best exploitation properties. This identification must be done on the basis of dynamical analysis, considering friction, wear and fatigue of material.

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